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Electromagnetic properties of the 'd'-wave superconductors

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Abstract

The 'd'-wave superconductor is mapped to a 2 + 1 Dirac fermion coupled to the Nambu–Goldstone phase. We integrate the Dirac fermion equation using the loop expansion and derive the Nambu–Goldstone phase action. This action is used to compute the ohmic conductivity and to estimate the Meissner penetration length.

1. Introduction

We study the electromagnetic properties of a two-dimensional 'd'-wave superconductor with a square Fermi surface [1]. A crucial ingredient of our calculation derives from the polarization diagram, given by

$$\Pi^{\mu,\nu}(\vec{q},\omega)A_\mu(\vec{q},\omega)A_\nu(-\vec{q},-\omega) \qquad \text{where } A_\mu(\vec{x},t) = \partial_\mu\alpha(\vec{x},t) - 2ea_\mu^{ext}$$

with α the Nambu–Goldstone phase of the superconductor. Due to the fact that the neutral-fermion excitations are described with the help of the Dirac equation in 2 + 1 dimensions, one obtains that the paramagnetic contribution of the polarization diagram is proportional to $q^2/\sqrt{(q^2-\omega^2-\mathrm{i}\epsilon)}$. This result has a number of consequences. We find that at $T = 0$ the penetration depth is given by

$$\delta = \delta_0\left(\frac{\Delta-\Delta_c}{\Delta_c}\right)^{-1/2} \qquad \Delta > \Delta_c$$

(Δ is the 'd'-wave superconducting order parameter). At finite temperature we obtain

$$\delta(T) = \delta_0(0)(1-T/\tilde{T}_0)^{-1/2}(1-T/T^*)^{-1/2} \qquad T^* = T_0(\Delta(0)/\Delta_c - 1).$$

The penetration depth is controlled by two parts. The first one, $(1-T/\tilde{T}_0)^{-1/2}$, is similar to the usual one and is controlled by the number of superconducting pairs. The second one is due to the paramagnetic polarization contribution. Recently a minimum penetration depth of $YBa_2Cu_3O_{7-\delta}$ films has been observed [9] and has been interpreted as evidence for low-energy Andreev bound states. Here we will not consider the bound states. Instead we will consider the penetration depth in the quasi-static limit. We find that for $|\omega/vq| > 1$, due to the branch cut induced by the polarization diagram, the penetration depth as a function of temperature has a minimum and the conductivity has an ohmic part.

We find that, since the Dirac spectrum is gapless, the charge–charge correlation has the form

$$\left(\frac{q}{\omega}\right)^2\left(1+\frac{\text{constant}}{\Delta}\frac{q^2}{\sqrt{q^2-\omega^2-\mathrm{i}\epsilon}}\right).$$

The plan of the paper is as follows. In section 2 we derive the effective action. We consider a square Fermi surface in the presence of a d-wave order parameter. We derive a Dirac action. We treat the Nambu–Goldstone phase as a gauge field. As a result we obtain a relativistic gauge theory in 2 + 1 dimensions. The integration of the relativistic fermions provides the polarization diagram which has the branch cut $q^2/\sqrt{(q^2-\omega^2-\mathrm{i}\epsilon)}$. Section 3 is devoted to the calculation of the effective action. Here we follow the standard strategy used at $T=0$ for deriving the Goldstone action [3]. Integration of the Goldstone field at $T=0$ enables us to obtain the electromagnetic response function in terms of the external fields. In section 4 we compute the penetration depth, conductivity, and charge–charge correlation. Section 5 is devoted to discussion.

2. The model for a square Fermi surface in the presence of a 'd'-wave order parameter in two dimensions

Electronic states near a square Fermi surface can be mapped onto one-dimensional quantum chains [1]. Starting from one electron per site one obtains a square Fermi surface. Two sets of chains arise, one for each axis of the square. When we introduce the 'd'-wave superconducting interaction, the Fermi surface is reduced to two pairs of Fermi surfaces, $\pm k_{F_1}$ and $\pm k_{F_2}$, with four nodal points [2]:

$$k_{F_1}=\left(\frac{\pi}{2a},\frac{\pi}{2a}\right)\qquad k_{F_2}=\left(\frac{\pi}{2a},-\frac{\pi}{2a}\right).$$

Around each nodal Fermi point one obtains a two-dimensional Dirac equation. When fluctuations of the 'd'-wave order parameter are included, our problem is equivalent to (2 + 1)-dimensional gauge theory for relativistic fermions.

Our starting point is the general representation for superconductivity introduced in reference [2]. The action for our model is

$$\begin{aligned}S=\int \mathrm{d}t\sum_{\vec r}\Bigg\{&\sum_{\sigma=\uparrow,\downarrow}\Bigg[C_\sigma^\dagger(\vec r,t)(\mathrm{i}\,\partial_t+ea_0^{ext}-E_F)C_\sigma(\vec r,t)\\&+t\sum_{\mu=x,y}(C_\sigma^\dagger(\vec r+\mathrm{d}\mu,t)\exp(\mathrm{i}ea_\mu^{ext})C_\sigma(\vec r,t)+\text{h.c.})\Bigg]\\&-\Bigg(\sum_{\mu=x,y}\Phi(\vec r,\vec r+\mathrm{d}\mu)[C_\uparrow^\dagger(\vec r,t)C_\downarrow^\dagger(\vec r+\mathrm{d}\mu,t)\\&-C_\downarrow^\dagger(\vec r,t)C_\uparrow^\dagger(\vec r+\mathrm{d}\mu,t)]+\text{h.c.}\Bigg)+\sum_{\mu=x,y}\frac{|\Phi(\vec r,\vec r+\mathrm{d}\mu)|^2}{2J}\Bigg\}.\end{aligned}\tag{1}$$

In the action S, $C_\sigma^\dagger$ and C_σ are the two-dimensional fermion fields. a_μ^{ext}, $\mu=0,x,y$ are the external vector potentials. $\Phi(\vec r,\vec r+\mathrm{d}\mu)$ and $\Phi^*(\vec r,\vec r+\mathrm{d}\mu)$ are the nearest-neighbour pairings (no on-site pairing). J is the superconducting coupling constant. The partition function of the action S given by equation (1) takes the form

$$Z=\int \mathrm{D}\Phi\,\mathrm{D}\Phi^*\,\mathrm{D}C_\uparrow^\dagger\,\mathrm{D}C_\uparrow\,\mathrm{D}C_\downarrow^\dagger\,\mathrm{D}C_\downarrow\,\mathrm{e}^{\mathrm{i}S}.\tag{2}$$

In the next step we expand the fermion fields around $\vec{K}_{F_1} = \vec{K}_{s=1}$ and $\vec{K}_{F_2} = \vec{K}_{s=2}$:

$$C_\sigma(\vec{r}) = \sum_{s=1,2} [\mathrm{e}^{\mathrm{i}\vec{K}_s\cdot\vec{r}} R_{s,\sigma}(\vec{r}) + \mathrm{e}^{-\mathrm{i}\vec{K}_s\cdot\vec{r}} L_{s,\sigma}(\vec{r})]. \tag{3}$$

$R_{s,\sigma}$ and $L_{s,\sigma}$ represent the right and left movers for each pair of axes;

$$\hat{e}_1 = \frac{\hat{x}+\hat{y}}{\sqrt{2}} \qquad \hat{e}_2 = \frac{\hat{x}-\hat{y}}{\sqrt{2}}$$

($\hat{x}$ and $\hat{y}$ are the unit vectors in the x- and y-directions). The superconducting order parameter $\Phi(\vec{r}, \vec{r}+\mathrm{d}\mu)$ can be parametrized in terms of a single Nambu–Goldstone excitation given by the phase $\alpha(\vec{r})$ and a gauge-invariant real field $\rho(\vec{r}, \vec{r}+\mathrm{d}\mu)$:

$$\Phi(\vec{r}, \vec{r}+\mathrm{d}\mu) = \mathrm{e}^{(\mathrm{i}/2)\alpha(\vec{r})}\rho(\vec{r}, \vec{r}+\mathrm{d}\mu)\mathrm{e}^{(\mathrm{i}/2)\alpha(\vec{r}+\mathrm{d}\mu)}.$$

$\alpha(\vec{r}) \equiv 2e\tilde{\alpha}(\vec{r})$ where e is the electric charge. The phase $\alpha(\vec{r})$ obeys the $\mathbb{Z}_2$-invariance condition $\alpha(\vec{r}) = \alpha(\vec{r}) + \pi\hbar$. Using this parametrization the integration measure is changed to $\int \mathrm{D}\rho\, \mathrm{D}\alpha$ with $\alpha(\vec{r})$ restricted to $0 \leqslant \alpha(\vec{r}) \leqslant \pi\hbar$. Using the fact that the integration measure is invariant under the transformation, we have

$$\begin{aligned} C_\sigma(\vec{r}) &= \mathrm{e}^{(\mathrm{i}/2)\alpha(\vec{r})}\tilde{C}_\sigma(\vec{r}) & C_\sigma^\dagger(\vec{r}) &= \tilde{C}_\sigma^\dagger(\vec{r})\mathrm{e}^{-(\mathrm{i}/2)\alpha(\vec{r})} \\ R_{s,\sigma}(\vec{r}) &= \mathrm{e}^{(\mathrm{i}/2)\alpha(\vec{r})}\tilde{R}_{s,\sigma}(\vec{r}) & R_\sigma^{s,\dagger}(\vec{r}) &= \tilde{R}_{s,\sigma}^\dagger(\vec{r})\mathrm{e}^{-(\mathrm{i}/2)\alpha(\vec{r})} \\ L_{s,\sigma}(\vec{r}) &= \mathrm{e}^{(\mathrm{i}/2)\alpha(\vec{r})}\tilde{L}_{s,\sigma}(\vec{r}) & L_\sigma^{s,\dagger}(\vec{r}) &= \tilde{L}_{s,\sigma}^\dagger(\vec{r})\mathrm{e}^{-(\mathrm{i}/2)\alpha(\vec{r})}. \end{aligned} \tag{4}$$

As a result of the transformation of equation (4), the action in equation (1) depends on the 'neutral' fields $\tilde{R}_{s,\sigma}$, $\tilde{R}_{s,\sigma}^\dagger$, $\tilde{L}_{s,\sigma}$, $\tilde{L}_{s,\sigma}^\dagger$, $\rho(\vec{r}, \vec{r}+\mathrm{d}\mu)$, and the effective gauge field:

$$ea_0 = ea_0^{ext} - \frac{1}{2}\partial_t\alpha \qquad e\vec{a} = e\vec{a}^{ext} - \frac{1}{2}\vec{\partial}\alpha. \tag{5}$$

In the remainder of this section we derive the continuum limit of the action in terms of the neutral fields $\tilde{C}_\sigma(\vec{r})$ and $\tilde{C}_\sigma^\dagger(\vec{r})$. Here we have to consider explicitly the kinetic term

$$H_0 = -t\sum_{\mu=x,y}\sum_{\vec{r}} \tilde{C}_\sigma^\dagger(\vec{r}+\mathrm{d}\mu)\mathrm{e}^{\mathrm{i}ea_\mu}\tilde{C}_\sigma(\vec{r}) + \mathrm{h.c.}$$

and the superconducting Hamiltonian:

$$\begin{aligned} H_{SC} = &\sum_{\mu=x,y}\sum_{\vec{r}} \rho(\vec{r}, \vec{r}+\mathrm{d}\mu)[\tilde{C}_\uparrow^\dagger(\vec{r})\tilde{C}_\downarrow^\dagger(\vec{r}+\mathrm{d}\mu) - \tilde{C}_\downarrow^\dagger(\vec{r})\tilde{C}_\uparrow^\dagger(\vec{r}+\mathrm{d}\mu) + \mathrm{h.c.}] \\ &+ \sum_{\mu=x,y}\sum_{\vec{r}} \frac{[\rho(\vec{r}, \vec{r}+\mathrm{d}\mu)]^2}{2J}. \end{aligned}$$

For the fermion we substitute the representation

$$\tilde{C}_\sigma(\vec{r}) = \sum_{s=1,2}(\mathrm{e}^{\mathrm{i}\vec{K}_s\cdot\vec{r}}\tilde{R}_{s,\sigma}(\vec{r}) + \mathrm{e}^{-\mathrm{i}\vec{K}_s\cdot\vec{r}}\tilde{L}_{s,\sigma}(\vec{r}))$$

and for the superconductor we consider the situation where the minimum occurs for the $\mathrm{d}_{x^2-y^2}$ case:

$$\rho(\vec{r}, \vec{r}+\mathrm{d}x) = -\rho(\vec{r}, \vec{r}+\mathrm{d}y). \tag{6}$$

We perform a derivative expansion, and rotate the coordinates:

$$\partial_1 = \frac{\partial_x + \partial_y}{\sqrt{2}} \qquad \partial_2 = \frac{\partial_x - \partial_y}{\sqrt{2}}$$

$$a_1 = \frac{a_x + a_y}{\sqrt{2}} \qquad a_2 = \frac{a_x - a_y}{\sqrt{2}}$$

and introduce the spinors

$$\psi_1 = \begin{pmatrix} \tilde{R}_{1,\uparrow} \\ \tilde{L}^{\dagger}_{1,\downarrow} \end{pmatrix} \qquad \psi_2 = \begin{pmatrix} \tilde{R}_{2,\uparrow} \\ \tilde{L}^{\dagger}_{2,\downarrow} \end{pmatrix}$$
$$\chi_1 = \begin{pmatrix} \tilde{R}_{1,\downarrow} \\ -\tilde{L}^{\dagger}_{1,\uparrow} \end{pmatrix} \qquad \chi_2 = \begin{pmatrix} \tilde{R}_{2,\downarrow} \\ -\tilde{L}^{\dagger}_{2,\uparrow} \end{pmatrix}. \tag{7}$$

Next we rewrite the action in the Lorentz-invariant form plus leading irrelevant terms. We introduce the γ-matrices:

$$\gamma^0 = \sigma_2 \qquad \gamma^1 = \mathrm{i}\sigma_1 \qquad \gamma^2 = \mathrm{i}\sigma_3$$
$$(\gamma^0)^2 = -(\gamma^1)^2 = -(\gamma^2)^2 = 1. \tag{8}$$

We introduce the velocities $v = 2\sqrt{2}t$, $\Delta(\vec{r}) = 2\sqrt{2}|\rho(\vec{r}, \vec{r} + \mathrm{d}\mu)|$, and use $\bar{\psi}_1 = \psi_1^{\dagger}\gamma^0$, $\bar{\psi}_2 = \psi_2^{\dagger}\gamma^0$, $\bar{\chi}_1 = \chi_1^{\dagger}\gamma^0$, $\bar{\chi}_2 = \chi_2^{\dagger}\gamma^0$. As a result, the action, equation (1), is replaced by the action $\tilde{S}$ in the continuum limit:

$$S = S^{(\Delta)} + S^{(\psi)} + S^{(\chi)}. \tag{9}$$

The form of $S^{(\psi)}$ is identical to that of $S^{(\chi)}$ since the action has no explicit dependence on the spin excitations.

$$S^{(\Delta)} = \int \mathrm{d}^2 r \int \mathrm{d}t \, \frac{\Delta^2(\vec{r})}{2J} \qquad J = \hat{J}\Lambda^{-3}. \tag{10}$$

The coupling constant J has the dimension Λ^{-3} since $\Delta(\vec{r})$ is dimensionless.

$$S^{(\psi)} = S_0^{(\psi)} + \delta S^{(\psi)}. \tag{11}$$

$S^{(\psi)}$ represents the relevant part of the action and $\delta S^{(\psi)}$ the leading irrelevant terms. The form of $S^{(\chi)} = S_0^{(\chi)} + \delta S^{(\chi)}$ is the same as that of $S^{(\psi)}$ with $\delta S^{(\chi)}$ being an irrelevant part similar to $\delta S^{(\psi)}$:

$$S_0^{(\psi)} = \int \mathrm{d}^2 r \, \mathrm{d}t \, \{\bar{\psi}_1\gamma^0(\mathrm{i}\,\partial_0 + eA_0)\psi_1 + v\bar{\psi}_1\gamma^1(\mathrm{i}\,\partial_1 + eA_1)\psi_1 + \Delta\bar{\psi}_1\gamma^2(\mathrm{i}\,\partial_2 + eA_2)\psi_1$$
$$+ \bar{\psi}_2\gamma^0(\mathrm{i}\,\partial_0 + eB_0)\psi_2 + \Delta\bar{\psi}_2\gamma^1(\mathrm{i}\,\partial_1 + eB_1)\psi_2 + v\bar{\psi}_2\gamma^2(\mathrm{i}\,\partial_2 + eB_2)\psi_2\}. \tag{12}$$

In equation (12) we have used the convention

$$va_1 = A_0 \qquad va_2 = B_0 \qquad \mathrm{i}[\Delta(\vec{r})]^{-1}\,\partial_2\Delta(\vec{r}) = A_2$$
$$B_2 = A_1 \equiv \frac{1}{v}a_0 \qquad \mathrm{i}[\Delta(\vec{r})]^{-1}\,\partial_1\Delta(\vec{r}) = B_1. \tag{13}$$

The irrelevant part is given by

$$\delta S^{(\psi)} = \int \mathrm{d}^2 r \, \mathrm{d}t \, \{-g_1[\bar{\psi}_1\gamma^0(-\partial_2^2 - 2\,\partial_1\,\partial_2)\psi_1 a_2 + \bar{\psi}_2\gamma^0(-\partial_1^2 - 2\,\partial_2\,\partial_1)\psi_2 a_1]$$
$$- g_2[\bar{\psi}_1\gamma^1(\mathrm{i}\,\partial_1)\psi_1(a_2)^2 + \bar{\psi}_2\gamma^2(\mathrm{i}\,\partial_2)\psi_2(a_1)^2]\}. \tag{14}$$

Dimensional analysis gives that v and $\Delta(\vec{r})$ are marginal and g_1 and g_2 are irrelevant.

$$v = \hat{v}\Lambda^0 \qquad \Delta(\vec{r}) = \hat{\Delta}(\vec{r})\Lambda^0 \qquad e = \hat{e}\Lambda^{(4-D)/2}$$
$$g_1 = \hat{g}_1\Lambda^{-D/4} \qquad g_2 = \hat{g}_2\Lambda^{2-D} \qquad D = d + 1 \tag{15}$$

where $\hat{g}_1$ and $\hat{g}_2$ are dimensionless and are given in terms of the dimensionless charge $\hat{e}$: $\hat{g}_1 = \hat{e}v/4$ and $\hat{g}_2 = \hat{e}^2 v/4$.

3. The effective action

In this section we will compute the effective action in terms of the gauge field $e\vec{a} = \vec{\partial}\alpha - e\vec{a}^{ext}$. We introduce the Green's function:

$$S_1(\vec{q},\omega) = \frac{\gamma^0\omega - \gamma^1(vq_1) - \gamma^2(\Delta q_2)}{-[\omega^2 - (vq_1)^2 - (\Delta q_2)^2]}$$
$$S_2(\vec{q},\omega) = \frac{\gamma^0\omega - \gamma^1(\Delta q_1) - \gamma^2(vq_2)}{-[\omega^2 - (vq_1)^2 - (\Delta q_2)^2]}. \tag{16}$$

In equation (16) we have replaced $\Delta(\vec{r})$ by Δ which is independent of $\vec{r}$. The integration of the fermion fields produces an effective action:

$$\Gamma(\Delta, a_0, \vec{a}) \stackrel{\text{def}}{=} \Gamma_0(\Delta, 0, 0) + \Gamma_2(\Delta, a_0, \vec{a})$$

where $\Gamma_0(\Delta, 0, 0)$ is the saddle-point action and where $\Gamma_2(\Delta, a_0, \vec{a})$ contains the Gaussian fluctuations.

$$Z = \int \mathrm{D}\alpha\, \mathrm{D}\Delta\, \mathrm{D}C_\uparrow^\dagger\, \mathrm{D}C_\uparrow\, \mathrm{D}C_\downarrow^\dagger\, \mathrm{D}C_\downarrow\, \mathrm{e}^{\mathrm{i}S} = \int \mathrm{D}\alpha\, \mathrm{D}\Delta\, \mathrm{e}^{\mathrm{i}\Gamma(\Delta,a_0,\vec{a})} \tag{17}$$

where

$$\Gamma_0(\Delta, 0, 0) = \int \mathrm{d}^2r \int \mathrm{d}t \left\{\frac{\Delta^2(\vec{r})}{2J} - \mathrm{i}\,2\,\mathrm{Tr}\log[S_1^{-1}] - \mathrm{i}\,2\,\mathrm{Tr}\log[S_2^{-1}]\right\}. \tag{18}$$

Δ is chosen using the condition $\partial\Gamma_0/\partial\Delta = 0$ and $\Gamma_2(\Delta, a_0, \vec{a})$ is given by

$$\begin{aligned}
\Gamma_2(\Delta, a_0, \vec{a}) = 2\int \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}^2q}{(2\pi)^2} \{&-\mathrm{i}g_2[\mathrm{Tr}(S_1(\vec{p},\Omega)\gamma^1 p_1)a_2(\vec{q},\omega)a_2(-\vec{q},-\omega) \\
&+ \mathrm{Tr}(S_2(\vec{p},\Omega)\gamma^2 p_2)a_1(\vec{q},\omega)a_1(-\vec{q},-\omega)] \\
&+ \frac{\mathrm{i}}{2}g_1^2[\mathrm{Tr}(\gamma^0 S_1(\vec{p},\Omega)(p_2^2 + 2p_1p_2) \\
&\times \gamma^0 S_1(\vec{p}+\vec{q},\Omega+\omega)((p_2+q_2)^2 + 2(p_1+q_1)(p_2+q_2)))a_2(\vec{q},\omega)a_2(-\vec{q},\omega) \\
&+ \mathrm{Tr}(\gamma^0 S_2(\vec{p},\Omega)(p_1^2 + 2p_1p_2) \\
&\times \gamma^0 S_2(\vec{p}+\vec{q},\Omega+\omega)((p_1+q_1)^2 + 2(p_1+q_1)(p_2+q_2)))a_1(\vec{q},\omega)a_1(-\vec{q},\omega)] \\
&+ e^2 v^\mu v^\nu \Pi^{(1)}_{\mu,\nu}(\vec{q},\omega)A_\mu(\vec{q},\omega)A_\nu(-\vec{q},-\omega) \\
&+ e^2 \tilde{v}^\mu \tilde{v}^\nu \Pi^{(2)}_{\mu,\nu}(\vec{q},\omega)B_\mu(\vec{q},\omega)B_\nu(-\vec{q},-\omega)\}.
\end{aligned} \tag{19}$$

In equation (19) the first two terms proportional to g_2 and g_1^2 represent the diamagnetic term induced by the irrelevant actions $\delta S^{(\psi)}$ and $\delta S^{(\chi)}$ (see equation (14)). In equation (19) v^μ and $\tilde{v}^\mu$ represent the velocity vector: $v^\mu = (1, v, \Delta)$ and $\tilde{v}^\mu = (1, \Delta, v)$. $\Pi^{(1)}_{\mu,\nu}(\vec{q},\omega)$ and $\Pi^{(2)}_{\mu,\nu}(\vec{q},\omega)$ represent the polarization diagrams:

$$\Pi^{(1)}_{\mu,\nu}(\vec{q},\omega) = \frac{\mathrm{i}}{2}\mathrm{Tr}[\gamma^\mu S_1(\vec{p}+\vec{q},\Omega+\omega)\gamma^\nu S_1(\vec{p},\omega)]$$
$$\Pi^{(2)}_{\mu,\nu}(\vec{q},\omega) = \frac{\mathrm{i}}{2}\mathrm{Tr}[\gamma^\mu S_2(\vec{p}+\vec{q},\Omega+\omega)\gamma^\nu S_2(\vec{p},\omega)]. \tag{20}$$

Following the method given in references [4] and [5] (see equation (39*a*) in reference [5]) we evaluate the polarization diagram for relativistic fermions in 2 + 1 dimensions. (In equations (18) and (19) we use the convention

$$\mathrm{Tr} \equiv \int \frac{\mathrm{d}\Omega}{2\pi} \int \frac{\mathrm{d}^2p}{(2\pi)^2}\, \hat{\mathrm{tr}}$$

where $\hat{\mathrm{tr}}$ stands for the γ-matrices.) The polarization diagrams are given by

$$\Pi^{(1)}_{\mu,\nu}(\vec{q},\omega) = \frac{\Gamma(2-3/2)}{(4\pi)^{3/2}} \int_0^1 \mathrm{d}x\ \sqrt{x(1-x)}\left[\frac{1}{v\Delta}\hat{\Pi}^{(1)}_{\mu,\nu}(\vec{q},\omega)\right] \tag{21}$$

where

$$\hat{\Pi}^{(1)}_{\mu,\nu}(\vec{q},\omega) = \frac{[\omega^2-(vq_1)^2-(\Delta q_2)^2]g^{\mu\nu}+v^\mu v^\nu q^\mu q^\nu}{[(vq_1)^2+(\Delta q_2)^2-\omega^2-\mathrm{i}\epsilon]^{-1/2}} \tag{22}$$

with $g^{\mu\nu}=0$ for $\mu\neq\nu$ and $g^{00}=-g^{\mu\mu}=1$. The expression for $\Pi^{(2)}_{\mu,\nu}(\vec{q},\omega)$ is similar to that for $\Pi^{(1)}_{\mu,\nu}(\vec{q},\omega)$ when we exchange q_1 with q_2. In equations (19)–(22) we have used for Δ the saddle-point value obtained from the condition $\partial\Gamma_0/\partial\Delta=0$. Next we evaluate the value of the saddle point Δ using $\Gamma_0(\Delta,0,0)$. Using equation (18) we obtain

$$\begin{aligned}
\Gamma_0(\Delta,0,0) &= [\Gamma_0(\Delta,0,0)-\Gamma_0(0,0,0)]+\Gamma_0(0,0,0)\\
&= -\mathrm{i}\,2\log\det[\mathrm{i}\gamma^0\,\partial_0+\mathrm{i}v\gamma^1\,\partial_1+\mathrm{i}\Delta\gamma^2\,\partial_2]+\mathrm{i}\,2\log\det[\mathrm{i}\gamma^0\,\partial_0+\mathrm{i}v\gamma^1\,\partial_1]\\
&\quad -\mathrm{i}\,2\log\det[\mathrm{i}\gamma^0\,\partial_0+\mathrm{i}\Delta\gamma^1\,\partial_1+\mathrm{i}v\gamma^2\,\partial_2]+\mathrm{i}\,2\log\det[\mathrm{i}\gamma^0\,\partial_0+\mathrm{i}v\gamma^2\,\partial_2]\\
&\quad +\frac{\Delta^2}{8J}+\text{constant}\\
&= \mathrm{i}\,2\int\frac{\mathrm{d}\Omega}{2\pi}\frac{\mathrm{d}^2p}{(2\pi)^2}\left\{\log\left[1-\frac{(\Delta p_2)^2}{\Omega^2-(vp_1)^2}\right]+\log\left[1-\frac{(\Delta p_1)^2}{\Omega^2-(vp_2)^2}\right]\right\}\\
&\quad +\frac{\Delta^2}{8J}+\text{constant}.
\end{aligned} \tag{23}$$

At $T=0$ we obtain

$$\begin{aligned}
\Gamma_0(\Delta,0,0,) = -2v\int_0^\Lambda\frac{\mathrm{d}p}{2\pi}\,p\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}&\left\{\left[\sqrt{\cos^2\theta+\left(\frac{\Delta}{v}\right)^2\sin^2\theta}-\sqrt{\cos^2\theta}\right]\right.\\
&\left.+\left[\sqrt{\sin^2\theta+\left(\frac{\Delta}{v}\right)^2\cos^2\theta}-\sqrt{\sin^2\theta}\right]\right\}+\frac{\Delta^2}{8J}+\text{constant}.
\end{aligned} \tag{24}$$

The saddle point Δ is obtained from the condition $\partial\Gamma_0/\partial\Delta=0$. A non-zero solution for Δ is given by

$$\frac{3\pi}{4\hat{J}/v} = \int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\left[\frac{\sin^2\theta}{\sqrt{\cos^2\theta+(\Delta/v)^2\sin^2\theta}}+\frac{\cos^2\theta}{\sqrt{\sin^2\theta+(\Delta/v)^2\cos^2\theta}}\right] \tag{25}$$

where J has been replaced by the dimensionless coupling constant $\hat{J}$: $J=\hat{J}\Lambda^{-3}$. The solution of equation (25) is such that when $\hat{J}\to 0$, $\Delta\to 0$. Therefore we conclude that at $T=0$ for $\hat{J}\neq 0$ we have $\Delta\neq 0$.

The value $\hat{J}=\hat{J}_1=(3\pi/2)v$ is special since we obtain the solution $\Delta/v=1$. Next we solve equation (25) in the limit $\hat{J}/v\to 0$ and find

$$\frac{\Delta}{v}\sim\left(\frac{2}{\pi}\right)^2\frac{\hat{J}}{3v}.$$

At $T\neq 0$ equation (25) is replaced by

$$\begin{aligned}
\frac{\pi}{4\hat{J}/v} = \int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\int_0^1\mathrm{d}y\ y^2&\left\{\sin^2\theta\tanh\left[y\frac{v\Lambda}{2K_BT}\epsilon(\theta)\right]\Big/\epsilon(\theta)\right.\\
&\left.+\cos^2\theta\tanh\left[y\frac{v\Lambda}{2K_BT}\tilde{\epsilon}(\theta)\right]\Big/\tilde{\epsilon}(\theta)\right\}
\end{aligned} \tag{26}$$

where $\epsilon(\theta)$ and $\tilde{\epsilon}(\theta)$ are given by

$$\epsilon(\theta) = \sqrt{\cos^2\theta + \left(\frac{\Delta}{v}\right)^2 \sin^2\theta}$$

$$\tilde{\epsilon}(\theta) = \sqrt{\sin^2\theta + \left(\frac{\Delta}{v}\right)^2 \cos^2\theta}.$$

For a fixed $\hat{J} \neq 0$ we compute the temperature T_0 for which $\Delta(T_0) = 0$. We find

$$K_B T_0 \sim \frac{v\Lambda}{\pi}\left(\frac{\hat{J}}{v}\right).$$

Using the value of the gap at $T = 0$:

$$\Delta(T=0) = \Delta(0) = \frac{4}{3\pi^2}\hat{J} \sim \frac{4}{3\pi}\frac{K_B T_0}{\Lambda}$$

we have for $T < T_0$,

$$\Delta(T) \sim \Delta(0)\left(\frac{T_0 - T}{T_0}\right).$$

Using the value of Δ we compute $\Gamma_2(\Delta, a_0, \vec{a})$. We separate equation (19) into two parts:,

$$\Gamma_2(\Delta, a_0, \vec{a}) \equiv \Gamma_2^{\text{diamagnetic}}(\Delta, a_0, \vec{a}) + \Gamma_2^{\text{polarization}}(\Delta, a_0, \vec{a}).$$

$\Gamma_2^{\text{diamagnetic}}(\Delta, a_0, \vec{a})$ is given by the irrelevant terms g_2 and g_1^2 in equation (19) and $\Gamma_2^{\text{polarization}}(\Delta, a_0, \vec{a})$ represents the second part described by equation (19) and equation (20). The diamagnetic part takes the form

$$\begin{aligned}
\Gamma_2^{\text{diamagnetic}}&(\Delta, a_0, \vec{a}) \\
&= 2\int \frac{d\omega}{2\pi}\int \frac{d^2q}{(2\pi)^2}\left\{\left[-\hat{g}_2\Lambda^{2-D}\int^{\Lambda}\frac{dp}{2\pi}\,p^2\int\frac{d\theta}{2\pi}\,\frac{\cos^2\theta}{\sqrt{\cos^2\theta + (\Delta/v)^2\sin^2\theta}}\right.\right. \\
&\quad \left.\left. + \frac{\hat{g}_1^2}{2v}\Lambda^{-D}\int^{\Lambda}\frac{dp}{2\pi}\,p^2\int\frac{d\theta}{2\pi}\frac{\sin^4\theta + \sin^2(2\theta)}{\sqrt{\cos^2\theta + (\Delta/v)^2\sin^2\theta}}\right]\vec{a}(\vec{q},\omega)\cdot\vec{a}(-\vec{q},-\omega)\right\} \\
&= \int\frac{d\omega}{2\pi}\int\frac{d^2q}{(2\pi)^2}\left\{-\frac{v}{2}K\left(\frac{\Delta}{v}\right)\left(\frac{1}{2}\,\vec{\partial}\alpha - \hat{e}\vec{a}^{ext}\right)_{\vec{q},\omega}\cdot\left(\frac{1}{2}\,\vec{\partial}\alpha - \hat{e}\vec{a}^{ext}\right)_{-\vec{q},-\omega}\right\}
\end{aligned} \tag{27}$$

where $K(\Delta/v)$ is given by

$$K\left(\frac{\Delta}{v}\right) = \frac{\Lambda^2}{12\pi}\int_0^{2\pi}\frac{d\theta}{2\pi}\left[2\frac{\cos^2\theta}{\sqrt{\cos^2\theta + (\Delta/v)^2\sin^2\theta}} - \frac{3}{10}\frac{\sin^4\theta + 4\sin^2\theta\cos^2\theta}{\sqrt{\cos^2\theta + (\Delta/v)^2\sin^2\theta}}\right]. \tag{28}$$

In obtaining equation (28) we have substituted $D = 3$ and have made the replacements

$$\hat{g}_2 = \hat{g}_1^2/v = \hat{e}^2 v/4 \qquad e = \hat{e}\Lambda^{(1-D)/2}.$$

We evaluate equation (28) and find for $\Delta/v \sim 1$ a power expansion in $[1 - (\Delta/v)^2]$:

$$K\left(\frac{\Delta}{v}\right) \sim \frac{\Lambda^2}{12\pi}\left\{1 - 0.04\left[1 - \left(\frac{\Delta}{v}\right)^2\right] + \cdots\right\}.$$

Equation (28) vanishes for $\Delta = \Delta_{min}$: $\Delta_{min}/v \ll 1$. Therefore we can express $K(\Delta/v)$ for small values of Δ as

$$K\left(\frac{\Delta}{v}\right) \simeq \frac{\Lambda^2}{12\pi}\left(\frac{\Delta - \Delta_{min}}{\Delta_{min}}\right).$$

This leads at finite temperature to the result

$$K\left(\frac{\Delta}{v}\right) = K\left(\frac{\Delta(0)}{v}\right)\left(\frac{T - \tilde{T}_0}{\tilde{T}_0}\right) \qquad \tilde{T}_0 \leqslant T_0 \tag{29}$$

where $\tilde{T}_0$ is the temperature at which $K(\Delta(\tilde{T}_0)/v) = 0$. Next we evaluate $\Gamma_2^{\text{polarization}}(\Delta, a_0, \vec{a})$ with the help of equations (21) and (22). We make the substitutions $B_2 = A_1 = (1/v)a_0$, $A_0 = va_1$, $B_0 = va_2$; we neglect the amplitude fluctuations and make the replacement $A_2 = B_1 = 0$ (see equation (13)):

$$\begin{aligned}
\Gamma_2^{\text{polarization}}(\Delta, a_0, \vec{a}) = \int \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \Bigg\{ & -\frac{v}{2}(\partial_1\alpha - 2\hat{e}a_1^{ext})_{\vec{q},\omega} \\
& \times \left[\frac{0.13(e/\hat{e})^2}{4\Delta} P_{11}(\vec{q},\omega)\right](\partial_1\alpha - 2\hat{e}a_1^{ext})_{-\vec{q},-\omega} \\
& - \frac{v}{2}(\partial_2\alpha - 2\hat{e}a_2^{ext})_{\vec{q},\omega}\left[\frac{0.13(e/\hat{e})^2}{4\Delta} P_{22}(\vec{q},\omega)\right](\partial_2\alpha - 2\hat{e}a_2^{ext})_{-\vec{q},-\omega} \\
& + \frac{1}{2v}(\partial_0\alpha - 2\hat{e}a_0^{ext})_{\vec{q},\omega}\left[\frac{0.13(e/\hat{e})^2}{4\Delta}\right](\partial_0\alpha - 2\hat{e}a_0^{ext})_{-\vec{q},-\omega} \\
& + \frac{0.13v}{4\Delta}(\partial_0\alpha - 2\hat{e}a_0^{ext})_{\vec{q},\omega} \\
& \times [P_{01}(\vec{q},\omega)(\partial_1\alpha - 2\hat{e}a_1^{ext})_{-\vec{q},-\omega} + P_{02}(\vec{q},\omega)(\partial_2\alpha - 2\hat{e}a_2^{ext})_{-\vec{q},-\omega}\Bigg\}
\end{aligned} \tag{30}$$

where the elements of the polarization matrix are

$$\begin{aligned}
P_{11}(\vec{q},\omega) &= \frac{(vq_1)^2 + (\Delta q_2)^2}{[(vq_1)^2 + (\Delta q_2)^2 - \omega^2 - \mathrm{i}\epsilon]^{1/2}} \\
P_{22}(\vec{q},\omega) &= \frac{(vq_2)^2 + (\Delta q_1)^2}{[(vq_2)^2 + (\Delta q_1)^2 - \omega^2 - \mathrm{i}\epsilon]^{1/2}} \\
P_{01}(\vec{q},\omega) &= \frac{-\omega q_1}{[(vq_1)^2 + (\Delta q_2)^2 - \omega^2 - \mathrm{i}\epsilon]^{1/2}} \\
P_{02}(\vec{q},\omega) &= \frac{-\omega q_2}{[(vq_2)^2 + (\Delta q_1)^2 - \omega^2 - \mathrm{i}\epsilon]^{1/2}} \\
\hat{P}_{00}(\vec{q},\omega) &\equiv P_{00}^{(1)}(\vec{q},\omega) + P_{00}^{(2)}(\vec{q},\omega) \\
P_{00}^{(1)}(\vec{q},\omega) &= \frac{(\Delta q_2)^2 - \omega^2}{[(vq_1)^2 + (\Delta q_2)^2 - \omega^2 - \mathrm{i}\epsilon]^{1/2}} \\
P_{00}^{(2)}(\vec{q},\omega) &= \frac{(\Delta q_1)^2 - \omega^2}{[(vq_2)^2 + (\Delta q_1)^2 - \omega^2 - \mathrm{i}\epsilon]^{1/2}}.
\end{aligned} \tag{31}$$

In the final step we add the diamagnetic contribution, equation (27), to the polarization part given by equation (30). We integrate out the Nambu–Goldstone field α and obtain the effective action as a function of the external fields. At $T \to 0$ we neglect the vortex contributions and consider only the Gaussian part of the field α:

$$\begin{aligned}\Gamma_{eff}(\Delta, a_0^{ext}, \vec{a}^{ext}) = \int \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \Bigg\{ &-\frac{1}{2} a_0^{ext}(\vec{q}, \omega) \\ &\times \left[\frac{q_1^2 K_{11}(\vec{q}, \omega) + q_2^2 K_{22}(\vec{q}, \omega)}{\omega^2} \right] a_0^{ext}(-\vec{q}, -\omega) \\ &- \frac{1}{2} a_1^{ext}(\vec{q}, \omega)[K_{11}(\vec{q}, \omega)] a_1^{ext}(-\vec{q}, -\omega) \\ &- \frac{1}{2} a_2^{ext}(\vec{q}, \omega)[K_{22}(\vec{q}, \omega)] a_2^{ext}(-\vec{q}, -\omega) \\ &- a_0^{ext}(\vec{q}, \omega) \left[K_{11}(\vec{q}, \omega) \left(\frac{q_1}{\omega} \right) \right] a_1^{ext}(-\vec{q}, -\omega) \\ &- a_0^{ext}(\vec{q}, \omega) \left[K_{22}(\vec{q}, \omega) \left(\frac{q_2}{\omega} \right) \right] a_2^{ext}(-\vec{q}, -\omega) \Bigg\} \end{aligned} \tag{32}$$

where

$$\begin{aligned} K_{11}(\vec{q}, \omega) &= \frac{ve^2}{4\pi} \left[R\left(\frac{\Delta}{v} \right) + \frac{1.6}{\Delta} P_{11} \left(\frac{\vec{q}}{\Lambda}, \frac{\omega}{\Lambda} \right) \right] \\ K_{22}(\vec{q}, \omega) &= \frac{ve^2}{4\pi} \left[R\left(\frac{\Delta}{v} \right) + \frac{1.6}{\Delta} P_{22} \left(\frac{\vec{q}}{\Lambda}, \frac{\omega}{\Lambda} \right) \right] \end{aligned} \tag{33}$$

$$R\left(\frac{\Delta}{v} \right) \stackrel{\mathrm{def}}{=} \frac{1}{12} K\left(\frac{\Delta}{v} \right) \left(\frac{\hat{e}}{e} \right)^2. \tag{34}$$

We remark that if $\vec{a}^{ext}$ is a transverse gauge field, the terms $a_0^{ext} a_1^{ext}$ and $a_0^{ext} a_2^{ext}$ in equation (32) vanish.

4. The electromagnetic response

Using equation (32) we will study the response of the 'd'-wave superconductor to the external fields. We will consider the penetration depth, charge–charge correlation, and the ohmic conductivity. We start the analysis with the penetration depth. For normal superconductors ('s' wave) the polarization diagram vanishes at $T = 0$. For the 'd'-wave case the polarization part gives rise to significant corrections to the penetration depth. Using equation (32) we compute the current $J_1 = -\partial \Gamma_{eff} / \partial a_1^{ext}$, $\vec{\nabla} \cdot \vec{a}^{ext} = 0$, and find

$$J_1(\vec{q}, \omega) = -K_{11}(\vec{q}, \omega) a_1^{ext}(\vec{q}, \omega) \tag{35}$$

where $K_{11}(\vec{q}, \omega)$ is given by equations (33) and (34). We will use equation (35) to compute the penetration depth. Due to the fact that $\Delta / v \neq 1$ and $P_{11}(\vec{q}, \omega) \neq P_{22}(\vec{q}, \omega)$, the penetration depth will depend on the direction. For simplicity we will assume that the two-dimensional 'd'-wave superconductor is rotated by 45° and that a current J_0 is applied along the $\hat{y}$-edge of the sample. As a result the magnetic field $\vec{B}(\vec{r})$ will be in the z-direction. Using the analysis given in reference [6] we find

$$B(\vec{r}) = -\frac{4\pi}{c} J_0 \int_{-\infty}^{\infty} \frac{\mathrm{d}^2 q_1}{2\pi} \frac{\mathrm{i} q_1 \mathrm{e}^{\mathrm{i} \vec{q}_1 \cdot \vec{r}}}{q_1^2 + 4\pi K_{11}(q_1, q_2 = 0, \omega)}. \tag{36}$$

Equation (36) is evaluated by contour integration. If the equation

$$q_1^2 + 4\pi K_{11}(q_1, q_2 = 0, \omega) = 0 \tag{37}$$

has no real solution, we have a Meissner effect. In order to solve equation (37) we substitute $\omega = 0$ and use the dimensionless charge $\hat{e}^2 = 4\pi/137$. Equation (37) takes the dimensionless form ($s = q_1/\Lambda$, $\omega = 0$)

$$s^2 + v\frac{4\pi}{137}\left[R\left(\frac{\Delta}{v}\right) + \frac{1.6}{\Delta/v}\frac{s^2}{\sqrt{s^2 - \mathrm{i}\epsilon}}\right] = 0. \tag{38}$$

We solve equation (38) in the limit $\epsilon \to 0$ and find that the condition for an imaginary solution for s is given by

$$\left(\frac{v\,4\pi}{137}\frac{1.6}{\Delta/v}\right)^2 - 4\left[\frac{v\,4\pi}{137}R\left(\frac{\Delta}{v}\right)\right] \leqslant 0.$$

This condition is equivalent to a critical value $\Delta = \Delta_c$ given by $\Delta_c/v \simeq 0.6/v$. This leads to the following penetration depth at $T = 0$:

$$\delta = \delta_0\left(\frac{\Delta - \Delta_c}{\Delta_c}\right)^{-1/2} \qquad \delta_0 = \frac{1}{\sqrt{ve^2R(\Delta/v)}}. \tag{39}$$

Using the fact that $\Delta = \Delta(T)$, we can estimate the penetration depth at $T \neq 0$:

$$\begin{aligned}\delta(T) &= \delta_0(0)\left(1 - \frac{T}{\tilde{T}_0}\right)^{-1/2}\left[\frac{\Delta(0)}{\Delta_c}\left(1 - \frac{T}{T_0}\right) - 1\right]^{-1/2} \\ &= \delta(0)\left(1 - \frac{T}{\tilde{T}_0}\right)^{-1/2}\left(1 - \frac{T}{T^*}\right)^{-1/2}\end{aligned} \tag{40}$$

where

$$T^* = T_0\left(\frac{\Delta(0)}{\Delta_c} - 1\right) \qquad T < T^* < \tilde{T} < T_0$$

$$\delta(0) \equiv \frac{1}{\sqrt{ve^2R(\Delta(0)/v)}}\left(\frac{\Delta(0)}{\Delta_c} - 1\right)^{-1/2}.$$

The new thing in equation (40) is the term $(1 - T/T^*)^{-1/2}$ induced by the paramagnetic polarization diagrams.

Next we compute the penetration depth in the quasi-static limit $\omega = \omega_0 \neq 0$ in the presence of finite elastic scattering $\epsilon = \epsilon_0 \neq 0$. The presence of the elastic scattering introduces an infrared cut-off for the momentum q_0. Under this condition the penetration depth (computed at a fixed q_0 and frequency ω_0) takes the form

$$\begin{aligned}\delta^{-1}(T) &= \delta_0^{-1}(0)\,\mathrm{Re}\left\{\left[\left(1 - \frac{T}{T_0}\right) - \frac{\mathrm{e}^{\mathrm{i}\phi/2}X(\omega_0/T_0)}{1 - T/T_0}\right]^{1/2}\right\} \\ &= \delta_0^{-1}(0)\left[\left(1 - \frac{T}{\tilde{T}_0}\right)^2 + \frac{X^2(\omega_0/T_0)}{(1 - T/T_0)^2} - 2\cos\frac{\phi}{2}\frac{X(\omega_0/T_0)}{(1 - T/T_0)}\right]^{1/4}\cos\frac{\alpha}{2}\end{aligned} \tag{41}$$

where

$$\begin{aligned}X\left(\frac{\omega_0}{T_0}\right) &\equiv \frac{(vq_0/\sqrt{\Lambda})^2}{[((vq_0)^2 - \omega_0^2)^2 + \epsilon_0^2]^{1/2}} \\ \tan\phi &= \frac{\epsilon_0}{(vq_0)^2 - \omega_0^2} \\ \tan\alpha &= \frac{-\sin(\phi/2)X(\omega_0/T_0)(1 - T/T_0)^{-1}}{(1 - T/\tilde{T}_0) - \cos(\phi/2)X(\omega_0/T_0)(1 - T/T_0)^{-1}}.\end{aligned} \tag{42}$$

Equation (41) has the interesting property that as a function of (ω_0/vq_0) we can change the temperature dependence of $\delta(T)$. For $\omega_0/(vq_0) \ll 1$ the first term is dominant in equation (41). As a result, when $T \to 0$ we find that the penetration depth increases with the temperature:

$$\delta(T) \sim \delta_0(0)\left(1 + \text{constant} \times \frac{T}{T_0}\right).$$

In the limit $|\omega_0^2 - (vq_0)^2| \sim \epsilon_0^2$, the second term is dominant, giving rise to a decrease of the penetration depth with the temperature. In this range of parameters we find qualitatively the same behaviour of $\delta(T)$ as given in references [9, 10].

In the last part we compute the conductivity and charge–charge correlation. The conductivity is obtained when we substitute $-\mathrm{i}\omega a_1^{ext}(\vec{q},\omega) = E_1^{ext}(\vec{q},\omega)$ in equation (35). As a result, we find the conductivity for $\Delta/v = 1$:

$$\sigma(\vec{q},\omega) = -\mathrm{i}\frac{K(\vec{q},\omega)}{\omega} = \frac{e^2 v}{4\pi}\tilde{\sigma}(\vec{q},\omega)$$

where

$$\tilde{\sigma}(\vec{q},\omega) = -\frac{\mathrm{i}}{\omega}\left[R\left(\frac{\Delta}{v}\right) + \frac{1.6}{\Delta}\frac{vq/\Lambda}{[1-(\omega/[vq])^2 - \mathrm{i}\epsilon]^{1/2}}\right]. \tag{43}$$

We write $\tilde{\sigma} = \tilde{\sigma}_R + \mathrm{i}\tilde{\sigma}_I$. $\tilde{\sigma}_R$ is the ohmic part of the conductivity. For finite elastic scattering $\epsilon = \epsilon_0$, the ohmic part $\tilde{\sigma}_R$ is given as a function of

$$z = \frac{\omega}{vq} \qquad \text{and} \qquad \tan\phi = \frac{\epsilon_0}{1-z^2}$$

as follows:

$$\sigma_R(z,\epsilon_0) = \frac{\hat{e}^2 v}{4\pi}\frac{1.6}{\Delta}\frac{\sin(\phi/2)}{z[(1-z^2)^2 + \epsilon_0^2]^{1/4}}. \tag{44}$$

Equation (44) shows that the normal part of the conductivity is finite at $\omega \to 0$ and $q \to 0$, such that $\omega/q \to 1$. Equation (44) shows that unlike the 's' wave, the 'd' wave has an ohmic conductivity.

The charge–charge correlation is computed from equation (32):

$$\begin{aligned}\langle\rho(\vec{q},\omega)\rho(-\vec{q},-\omega)\rangle &= \frac{q_1^2 K_{11}(\vec{q},\omega) + q_2^2 K_{22}(\vec{q},\omega)}{\omega^2}\\ &\overset{\Delta/v\sim 1}{=} \left(\frac{q}{\omega}\right)^2\left[R\left(\frac{\Delta}{v}\right) + \frac{1.6}{\Delta}\frac{(vq/\sqrt{\Lambda})^2}{\sqrt{(vq)^2 - \omega^2 - \mathrm{i}\epsilon}}\right].\end{aligned} \tag{45}$$

Equation (45) shows that instead of a regular pole, we have a branch cut.

5. Discussion

The 'd'-wave superconductor has been mapped to an effective Nambu–Goldstone action. The new thing in the 'd'-wave action is the presence of the paramagnetic polarization which has a branch-cut form. Due to the branch-cut form the conductivity and the penetration depth are strongly renormalized. Using this action we have computed the ohmic conductivity and the microwave penetration depth. We find an upturn in the penetration depth, as a function of temperature, which might be relevant to recent experiments [9].

References

[1] Luther A 1994 unpublished
[2] Balents L, Fisher M P A and Naya C 1998 *Int. J. Mod. Phys.* B **12** 1038
[3] Weinberg S 1996 *The Quantum Theory of Fields* vol 2 (Cambridge: Cambridge University Press) section 21.6
[4] Weinberg S 1996 *The Quantum Theory of Fields* vol 1 (Cambridge: Cambridge University Press) section 11.2
[5] Jackiw R and Templeton S 1981 *Phys. Rev.* D **23** 2291
[6] Fetter A L and Walecka J D 1971 *Quantum Theory of Many-Particle Systems* (New York: McGraw-Hill) ch 49
[7] Dolan L S and Jackiw R 1974 *Phys. Rev.* D **9** 3320 (see especially pp 3334, 3335)
[8] Schrieffer J R 1964 *Theory of Superconductivity* (Reading, MA: Addison-Wesley)
[9] Walter H, Prusseit W, Semerad R, Kinder H, Assmon W, Huber A, Barkhandt H, Rainer D and Sauls J 1998 *Phys. Rev. Lett.* **80** 3598
[10] Baresh Y S, Kalerkoo M S and Kurkijarri J 2000 *Preprint* cond-mat/0005010